9/10

Very nice format!

The α assignment

Fundamentals of Nuclear Physics
Department of Nuclear Engineering
Universitat Politècnica de Catalunya

Abstract

This paper is a result of research done in response to the alpha decay assignment in the Fundamentals of Nuclear Physics course. It provides understanding of several phenomena occurring in connection to the alpha decay. Among these for example understanding the Quantum Tunneling effect, conservation of energy and momentum equations. Apart of these, also properties of several alpha emitters are listed and conclusions are made of these properties.

Keywords—alpha decay; Quantum Tunneling Effect; alpha emitters; conservation of energy and momentum

I. Introduction

To understand the behavior of fission products, several phenomena related to reactor physics, it is necessary to quantify the behavior of radioactive materials. One of which is the alpha decay. When a radioactive material releases an alpha particle ($^4_2He^{2+}$), we is that it undergoes the alpha decay. This paper is answering several questions and explaining main properties connected with this process.

in fact most alpha emitters have A>150

II. Alpha Decay

As an alpha decay we understand an emission of alpha particles (helium nuclei) from a heavy nucleus (mass number A>106), which can be represented by either the $^4_2He^{2+}$sign or the Greek letter α. During an alpha emission, the unstable nucleus ejects an alpha particle and the atomic number is then reduced by 2 and the mass number by 4. It is a quantum tunneling process.

For example:

$$^{234}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + {}^{4}_{2}\alpha + \gamma + \mathrm{KE}$$

Figure 1: Alpha decay example. [1]

III. Quantum Tunneling Effect

In the classical mechanics it is impossible for an object to pass through the potential barrier. It can be imagined as on a figure 1. A red ball is moving from rest at the edge of a hill. In the classical mechanics it cannot reach height greater than "h".

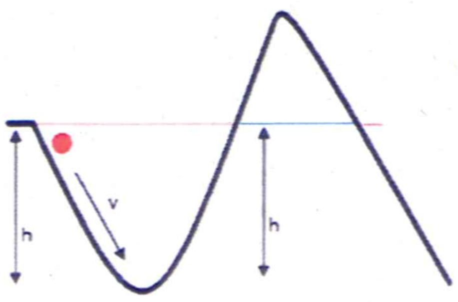

Figure 2: Potential barrier. (Self-made)

However imagine the ball being for example an alpha particle. In quantum mechanics there is a non-zero probability of the particle appearing on the other side of the hill. This phenomena is known as a quantum tunneling effect. In respect of the alpha decay, there is a non-zero probability of an alpha particle moving from the center of the nuclei to the outside of it. This is possible due to the fact that a particle behaves both as a particle and a wave. This property is called a wave-particle duality.

The probability of the QTE occurring is known as the transmission coefficient T.

$$T = \exp(-\frac{2}{\hbar}\int_{R_0}^{r_1}(2m|E - V(r)|)^{\frac{1}{2}}dr)$$

Where R0 is the atomic radius, E is the energy of the alpha particle, and r1 is the radius at which E = V(r). [1]

IV. Conservation of Energy and Momentum

The particle undergoing an alpha decay is called the parent particle and the result of the decay is called the daughter particle. During the emission the parent particle is split into an alpha and daughter particle and a certain amount of energy is then released.

$$mass\ difference = mass\ parent - (mass\ daughter + mass\ alpha)$$

For the conservation of momentum we consider the parent particle to be stationary.

$$M_D \cdot V_D = m_\alpha \cdot v_\alpha$$

If we square both sides and rearrange it, we can get:

$$\frac{E_{k_\alpha} + E_{k_D}}{E_{k_\alpha}} = \frac{m_\alpha + M_D}{M_D} = \frac{Total\ available\ energy}{E_{k_\alpha}} = \frac{M_P}{M_D}$$

Inverting gives us the maximum kinetic energy of the alpha particle in terms of total available energy.

$$E_{k_\alpha} = \left(\frac{M_D}{M_P}\right) \cdot Total\ available\ energy$$

The relativistic conservation of energy:

How are energies of alpha particles?

$$Q_\alpha = (M_P - M_\alpha - M_D)c^2$$

Where Q_α is the energy equivalent to the difference in masses between the initial and final states.

The non-relativistic conservation of energy is following:

$$E = \frac{1}{2}mv^2 Q_\alpha \ /\cdot 2m$$

There are more simple equations than these!

$$2mE = m^2v^2$$

The non-relativistic momentum is $p = mv \rightarrow 2mE = p^2$. We also know that $p_\alpha = p_D$so we get $2m_\alpha E_\alpha = 2m_D E_D$ and from that $E_D = \left(\frac{m_\alpha}{m_D}\right) E_\alpha$.

We can use the non-relativistic approximation, considering the mean velocity of an alpha particle to be approximately $2.015 \cdot 10^7 ms^{-1}$and its mass $6.64465675 \cdot 10^{-27} kg$. Then the Lorentz's factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is equal to 1.0022.

[2]

V. Sample of Alpha Emitters

In this part, some of main emitters that are appearing inside the nuclear reactor are listed, as well as some of their parameters.

Table 1: List of alpha emitters and their properties. Source: JEFF.3.1.1

Alpha Emitter	Atomic Number	Mean Gamma Energy [keV]	Alpha Energy [keV]	Recoil Energy [keV]
americium-241	95	22.4085	5465.1675	92.2776
plutonium-236	94	0.0572	5753.1411	99.2376
uranium-238	92	0.0530	4187.0737	71.6054
thorium-232	90	0.1452	4007.0901	70.3343
radon-222	86	0.3880	5489.2334	100.7780
polonium-210	84	0.0097	5304.4399	103.0752

Table 2: List of alpha emitters and their properties. Source: JEFF.3.1.1

Alpha Emitter	Uncertainty in Effective Q [%]	Calculated Q Value [keV]	Deviation of Calculated Q Value [%]
americium-241	0.0021	5624.0366	0.2443
plutonium-236	0.0014	5867.3486	-0.0047
uranium-238	0.0703	4270.0786	0.0003
thorium-232	0.0980	4091.7017	-0.2622
radon-222	0.0020	5590.4121	-0.0020
polonium-210	0.0013	5407.5249	-0.0014

Table 3: List of alpha emitters and their properties. Source: JEFF.3.1.1

Alpha Emitter	Atomic Number	Mean Gamma Energy [keV]	Alpha Energy [keV]	Recoil Energy [keV]
erbium-152	**68**	**0**	**4371.913**	**118.4515**
uranium-233	92	0.2105	4819.896	84.2311
curium-244	96	0.0164	5796.7568	96.6508
berkelium-247	97	113.1	5565.502	91.8219

Table 4: List of alpha emitters and their properties. Source: JEFF.3.1.1

Alpha Emitter	Uncertainty in Effective Q [%]	Calculated Q Value [keV]	Deviation of Calculated Q Value [%]
erbium-152	x	x	x
uranium-233	0.0244	4912.9492	-0.0825
curium-244	0.009	5903.7485	-0.0319
berkelium-247	x	x	x

According to the Lund/LBNL Nuclear Data Search website, the alpha energies for all alpha emitters are in a range between 1830 keV for 144-Nd and 11740 keV for 266-mMt. However most are in a range between 3-7 keV.

Another interesting observation among alpha emitters are those with atomic mass under 150u. For example that is 148-Gd, 149-Sm or 144-Nd. If we were to calculate the Q factor (chapter IV), we could see that these elements have it negative for the alpha emission.

Focusing on a specific parent nuclide, Americium-241, the term "mono-energetic" is explained. The definition of a mono-energetic alpha particles is following. *The amount of energy released is the same for each decay, and the two decay products can only share the energy one way. We say that the released alpha particle is mono-energetic.* [3]

Considering that according to table 5, 85% of alphas emitted by 241-Am are of 5485680eV and 13% of 5442980eV. These values are close enough to be considered as mono-energetic.

I do not see that alphas of Am-241 are "monoenergetic"!

Table 5: 241Am alpha emitters. Source: JEFF.3.1.1

E [eV]	ΔE [eV]	I [%]
5388400	200	1.65
5442980	140	13.1
5485680	120	84.4
5442980	140	13.1
5485680	120	84.4

References

[1] Liu, Lulu, and Pablo Solis. "The Quantum Mechanics of Alpha Decay." (2007): n. pag. *MIT*. Web. 23 Sept. 2014. <http://web.mit.edu/lululiu/Public/pixx/not-pixx/alpha.pdf>.

[2] Lewis, E. E. "1.7." *Fundamentals of Nuclear Reactor Physics*. Amsterdam: Elsevier/Academic, 2008. N. pag. Print.

[3] Siegel, Peter. "Nuclear Decays." *Nuclear Decays* (n.d.): n. pag. *Peter Siegel - Professor Emeritus*. California State Polytechnic University. Web. 24 Sept. 2014.